Review 26 Tests for Convergence

1. n^{th} term test for divergence

If a_n does not converge to 0, $\sum^{\infty} a_n$ diverges

2. geometric $\sum ar^{n-1} = a + ar + ar^2 \ldots$

$\sum \frac{1}{2^n}$ converge $|r|<1$, series converges to $\frac{a}{1-r}$

diverges $|r| \geq 1$

3. p-series $\sum \frac{1}{n^p}$ converge $p>1$

diverge $p \leq 1$

$\sum \frac{1}{n^2}$

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4 ratio test $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$ converge $L<1$, diverge $L>1$, ?? $L=1$

use on factorials, exponentials

5. integral test

$\sum_{n=1}^{\infty} a_n$ & $\int_k^{\infty} a_x \, dx$ behave the same

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6. Direct comparison

If $0 \leq a_n \leq b_n$ & $\sum^{\infty} b_n$ converges

then $\sum^{\infty} a_n$ also converges

If $0 \leq a_n \leq b_n$ & $\sum^{\infty} a_n$ diverges

then $\sum^{\infty} b_n$ also diverges

7. Limit comparison test

If $\lim_{n\to\infty} \frac{a_n}{b_n}$ = positive number (a_n & b_n grow at same rate)

then $\sum a_n$ & $\sum b_n$ behave the same

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8. Alternating Series Test (AST)

if
1. signs alternate
2. $|a_{n+1}| < |a_n|$ (terms get smaller)
3. terms go to 0 $a_n \to 0$

then $\sum a_n$ converges

remainder theorem

$a_1 - a_2 + a_3 - a_4 \ldots a_n$

remainder (error) $< |a_{n+1}|$

if $a_{n+1} > 0$ underest.

if $a_{n+1} < 0$ overest.

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9. absolute convergence: $\sum |a_n|$ converges

if $\sum |a_n|$ converges then $\sum a_n$ must converge

10. Conditional Convergence

$\sum |a_n|$ diverges, $\sum a_n$ converges

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots$

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Ex 1. which converge?

I $\sum \frac{1}{\sqrt{n}}$ p series, $p = \frac{1}{2}$ diverges

II $\sum \frac{2^n}{n!}$ converges $\lim_{n\to\infty} \left| \frac{2^{\cancel{n+1}}}{(n+1)\cancel{!}} \cdot \frac{\cancel{n!}}{\cancel{2^n}} \right| = 0 < 1$

diverges

III $\sum \frac{1}{\ln(n)} > \sum \frac{1}{n}$ harmonic, diverges

IV $\sum \left(\frac{n+1}{n}\right)^n$ $\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = e$

diverges by n^{th} term test

seq. converges to e

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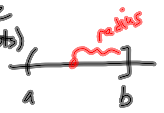
Interval of convergence

(must also consider endpts)



Radius of convergence

don't worry about endpts

use ratio test

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Find the ioc $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n \cdot 2^n}$

$$\lim_{n\to\infty} \left| \frac{(x-1)^{\cancel{n+1}}}{(n+1)\,2^{\cancel{n+1}}} \cdot \frac{n \cdot \cancel{2^n}}{\cancel{(x-1)^n}} \right| = \frac{|x-1|}{2} < 1$$

endpts $x = 3$ $|x-1| < 2$

$\sum \frac{2^n}{n \cdot 2^n} = \sum \frac{1}{n}$ $-2 < x-1 < 2$

harmonic diverges $-1 \le x < 3$

$x = -1$ $\sum \frac{(-2)^n}{n \cdot 2^n}$ converges AST

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